

Memorandum M-2828

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Digital Computer Laboratory
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

SUBJECT: BIWEEKLY REPORT, MAY 16, 1954

To: Jay W. Forrester

From: Scientific and Engineering Computation Group

1. MATHEMATICS, CODING AND APPLICATIONS1.1 Introduction

During the period covered by this report 378 coded programs were run on the time allocated to the Scientific and Engineering Computation (S&EC) Group. These programs represent part of the work that has been carried on in 35 of the problems that have been accepted by the S&EC Group. Progress on 25 of these problems is given below in terms of programming hours, minutes of computer time, and progress reports as submitted by the programmers in question.

The revised comprehensive system (CS II), described in previous biweekly reports, has been operating successfully (see problem #100). A memorandum (M-2798) has been issued describing tape room procedures for CS II. A memorandum is now being completed to describe for the programmer the changes and additions that have gone into CS II. These two memos together with M-2741, which was written primarily for the machine operators, should provide the information necessary for the routine adoption of CS II.

Three new problems were initiated during this period. Problem #176 is studying a general traffic problem with particular application to the provision of connectors in automatic telephone exchanges. This work is being carried on in cooperation with B. Marrows of the Electrical Engineering Department. Problem #183 is being carried out by H. Lin of the Aeroelastic Lab. with D. Sternlight of the Mathematics Department to establish a lethal criterion for airplane structures upon encountering a blast gust. Problem #184 involves the evaluation of various two-dimensional integrals that arise in the determination of the amplitude of electron scattering from hydrogen. This study is being carried out by M.C. Newstein in partial fulfillment of the requirements for a PhD degree in Physics.

The routines developed in Problem #147 by Dr. D. J. Howarth of the MIT Solid State and Molecular Theory Group are being used by Dr. F. Herman of R.C.A., Princeton to solve 52 secular equations of orders varying up to 20.

D.T.Ross of the Servomechanisms Laboratory has completed various power spectrum runs using his improved procedures (problem #171). These results will be incorporated in his S.M.Thesis.

Most of the staff members of the S&EC Group attended the symposium on Automatic Coding for Digital Computers sponsored by the Navy Mathematical Computing Advisory Panel on May 13 and 14. At this meeting Professor C.W.Adams described the Comprehensive, Summer Session, and Algebraic Systems.

1.2 Programs and Computer Operation

100. Comprehensive System of Service Routines, developed by the S&EC Group at the Digital Computer Laboratory for the input conversion of suitably prepared punched paper tapes. When so requested, these routines automatically provide a program with suitable programmed arithmetic, cycle-counting, and output facilities.

:DCL Staff: Arden, 41 hours; Best, 43 hours; Combelic, 33 hours; Demurjian, 14.25 hours; Denman, 27 hours; Frankovich, 38 hours; Helwig, 43 hours; Kopley, 7 hours; Porter, 4 hours; Siegel, 36 hours; WWI, 483 minutes

The S&EC comprehensive system of utility tapes has been in successful operation during the past biweekly period.

The following modifications to the system are now being discussed:

- (1) provisions for displaying post-mortem results on the scope;
- (2) recording post-mortem results in terms of floating addresses;
- (3) methods for reducing the time required to obtain automatic output requests.

Procedures for conveniently adding and checking new blocks and programs for the system are also being considered.

Staff

Descriptions of the adaptation and recording programs and the PA blocks are being written for the CS manual.

Helwig

It is planned to write a program which will make it possible to transfer blocks from one magnetic tape unit to another. (A block is a program recorded on magnetic tape which is properly identified so that it can be found and read in by the input program.) Since it is expected that future extensions of the CS II system will be tested by recording blocks on magnetic tape unit 2, this program could serve to transfer the tested blocks to magnetic tape unit 0. It will also make the insertion or lengthening of a block less time consuming.

Best

- 101 C. Optical Properties of Thin Metal Films on transparent backings are determined and printed out automatically by this program; the input data consist of the observed reflection and transmission coefficients, the index of the backing, the wavelength, and the sample thickness. The program calculates by means of an iterative procedure and prints out the index of refraction and the absorption coefficient of the film, the rate of variation of these constants with reflection and transmission, and the film's conductivity and dielectric constant.
 :for Professor L. Harris, Chemistry Department, Dr. A.L.Loeb
 :by Dr. A.L.Loeb (DIC), 20 hours; Richmond, 35 hours
 :DCL: WWI, 117 minutes

The present Principal Program contains a provision for having WWI make a first estimate of optical constants by using approximate equations. This provision appeared to have been tested with old parameter tapes for which results were already known. It was not realized at the time that since these parameter tapes contain the indication START AT 126^d the approximate program was actually short circuited out. Changing to iSTART AT 1200 produced new sets of optical constants which fit the experimental results as well as the old sets. It has long been realized that there are two sets of optical constants corresponding to one set of reflection and transmission data. One of these sets has no physical significance. In the approximate program either of these sets could be obtained; which one is obtained depends on the sign chosen for a square root. The physically significant set corresponds to the positive square root on one side of an absorption band, to the negative root on the other side of the absorption band. The non-significant set often has very large values, and caused overflows in the production run. The approximate program had been arbitrarily limited to one of the sets, which in production appeared to be the wrong one. Provision has now been made to select either one of the two sets, and this program is being tested now.

The accidental discovery that after the program tape has been read in the principal program can be used either with or without the first approximation provision simply by indicating iSTART AT 1200 or START AT 126 after the particular parameter under consideration is a very useful one. When START AT 126 is used, the parameter tape should contain a first estimate of optical constants; when iSTART AT 1200 is used this first estimate may be omitted.

- 106 C. MIT Seismic Project is concerned with the development of methods for locating deep reflections from underground strata in seismic prospecting. The basic method is one of prediction by means of an optimum linear operator.
 :for Professor P.M.Hurley, Geology and Geophysics;
 Professor G. Wadsworth, Mathematics Department
 :by E.A.Robinson(Res. Assoc.), Briscoe, 36 hours; Simpson, 20 hours; Treitel, 15 hours; Walsh, 25 hours
 :DCL: WWI, 281 minutes

During the past two weeks the various computation projects under immediate consideration by the group have proceeded as follows:

A program which generates artificial seismograms of a controlled statistical nature has been written and tested by one of the new members of the group. This is to be used in conjunction with a study of filtering behaviour under controlled conditions. One aspect of this study has been the result of overlapping wavelets for which the computational results have just returned.

The experiment on the operator criterion to be tested as mentioned in the April 19 biweekly has not yet reached completion due partially to a tape mix up in the Whirlwind subroutine library.

The study of non-stationary properties of seismograms has been directed in the last two weeks towards earthquake records. It has shown that various phases of body and surface waves may be detected in the nature of certain highly controversial phases. A future project on the factorization of the spectrum has required adjustments in existing correlation programs. These adjustments were also necessary for the study of the coherency matrix. Programming errors have slowed these adjustments and these projects are not completed.

- 119 C. Spherical Wave Propagation produced by the sudden release of a spherical distribution of compressed air in the atmosphere is being studied by numerical means. This involves replacing a set of non-linear hyperbolic partial differential equations in 2 independent and 2 dependent variables by a set of difference equations written along characteristics. An iterative procedure is used to solve these equations.
 :for Professor C.C.Lin, Mathematics Department
 :by A. Ralston (CMMC), 10 hours
 :DCL: WWI, 48 minutes

The problem has been slightly modified to eliminate the necessity of estimating an indeterminate quantity. This modification has been programmed and preliminary results have been obtained. A production run is slated for next week.

- 120 D. The Aerothermopressor - This problem is concerned with the development of a hot, high velocity gas stream by means of evaporative cooling. The analytical investigation being carried out on Whirlwind I involves the step-by-step solution of seven simultaneous non-linear differential equations which describe the thermodynamic and dynamic behavior of the compressible flow within the Aerothermopressor.
 :for Professor A.H.Shapiro, Department of Mechanical Engineering
 DIC 5-6985 , ONR N5ori-07878
 :by B.D.Gavril(DIC), 80 hours
 :DCL: WWI, 166 minutes

During the last two biweekly periods, computations relative to aerothermopressor performance for various types of Mach Number variations have continued. In addition, several new programs of considerable importance have been written.

Calculations of performance with prescribed gas stream properties, such as Mach Number or temperature, have in the past led to extremely large rates of increase of duct cross-sectional area near the end of the process. Such behavior is undesirable both from a point of view of controlling truncation error and from a consideration of the one-dimensional nature of the analysis. This situation has been avoided by an automatic procedure which detects such conditions and continues the process with a fixed rate of increase of duct diameter corresponding to a given cone angle, usually 6° . The problem of diffusion to low velocity is therefore accomplished with a minimum of truncation error while at the same time duplicating what is attainable in practice.

One of the most troublesome obstacles in calculations of the aerothermopressor process has been instability of the difference equations associated with errors in the liquid temperature. To continue the calculations beyond a value of humidity corresponding to roughly 70% of that when saturated, always required a drastic reduction in increment size. The objection to such a requirement is obvious from a point of view of computer time. Through a modified calculation procedure, this difficulty has been eliminated, and it is now possible to continue the calculations to saturation without decreasing the increment size. This is very significant since it is now possible to iterate on the whole process within a reasonable length of time.

The difficulty associated with the liquid temperature may be traced to the fact that the vapor pressure of the liquid, that is, the driving force for evaporation, is very sensitive to changes in liquid temperature; an error of 1% in the liquid temperature results in an error in vapor pressure of roughly 15% in the range of interest. This is further magnified in the rate of evaporation which depends upon a small difference of two large quantities, one of which is the vapor pressure of the liquid. Errors in liquid temperature can be magnified 70 to 1 in the rate of evaporation. In the calculation procedure previously used, the change in liquid temperature was calculated from energy considerations which also involved a small difference of two numbers, the rate of evaporation being contained in one of them. The futility of the procedure is apparent; errors in liquid temperature could be magnified on the order of 1500 to 1 in the value of the increment of liquid temperature!

This severe sensitivity has been eliminated by modifying the calculation procedure to avoid calculation of the rate of evaporation directly from the vapor pressure relations which depend so strongly upon the liquid temperature. These gas stream properties control the evaporation process, and the actual liquid temperature closely follows the wet-bulb temperature of the stream, which can be calculated by suitable iteration procedures. If the liquid temperature is taken to be the wet-bulb temperature, approximately, the rate of evaporation can be calculated to within a few percent from energy considerations which require that the

rate of heat transfer be equivalent to the rate of change of sensible heat of the droplet plus that required for evaporation.

This is substantially an outline of the procedure used. Several constant area calculations carried out a year ago have been repeated using this procedure and the agreement was found to be excellent, with the current procedure allowing the calculations to continue to completion.

Most of the programming time, however, was concerned with a calculation procedure for systematically determining optimum performance of the aerothermopressor. This program has been completed and testing will begin during the next biweekly period. This program, which numerically determines the best polynomial solution to the process for certain boundary conditions, will be discussed in the next biweekly report.

- 123 C. Earth Resistivity measurements are used to calculate the Slichter kernel function which, in special cases, can be analyzed to give the actual distribution of resistivity. The method involves least-square fitting a set of polynomials to the measured surface-potential function and integrating the product of this set and the zero-order Bessel function.
: for P.M.Hurley, Department of Geology and Geophysics, DIC 5-6915
: by K. Vozoff(Res. Assist. CMMC), 50 hours
:DCL: WWI, 112 minutes

Since the last report, two programs, one for integrating the surface potential to find the Slichter Kernel, and the other for analyzing the kernel, have been run successfully.

Trouble on the former had arisen from the recurrence calculation of Bessel functions of orders greater than nine, when roundoff accumulation caused the calculation to diverge. This was solved by reverting to a series calculation with a "smallest-term" criterion for cutting off the series. A minor mistake in programming was found in the last performance and has been corrected. This will be tested for several sets of data before completion.

The kernel analysis program has been run for ten steps using two different step size factors. These factors are used to multiply the changes in the parameters indicated by the matrix solution and are required because of the non-linearity of the kernel function. The first factor used was 0.1, which was found to be too large since the solutions diverged after three steps. The second factor used was 0.01. Convergence here was definite, but rather slow, and 0.025 is now being tested. This program will also be tested for several sets of data.

When both programs have been satisfactorily tested they will be tied together to give an analysis directly from surface potential data.

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131. The Training of New Personnel, Tours and Demonstrations are among those activities included in this problem. Generally speaking, any approved staff problem relating to training and/or demonstrations is considered to be in this category.
:DCL Staff: Demurjian, 3.25 hours; Denman, 1 hour; Kopley, 32 hours; WWI, 65 minutes

Movie: Shooting of the film "Making Electrons Count" will be completed by May 20. It is hoped that the film will be ready to be shown at the ACM meeting at Michigan University on June 23-25, 1954.

Tours: Twenty-six people attended the monthly tour which included Flexowriter and computer demonstration, tour of Whirlwind installations, and an informal twenty-minute talk.

- 132 C. Subroutines for the Numerically Controlled Milling Machine are being revised and tested. The set of subroutines facilitates programming of the computations involved in the preparation of numerical data used to control the milling machine. The subroutines involve routine numerical and logical operations.
:for J.O.McDonough, Servomechanisms Laboratory, DIC 6873
:by J.H.Runyon (E.E.Res. Assist.), 20 hours
:DCL: WWI, 14 minutes

A revised routine for reading milling machine tape was written but has not yet been tested. Several more errors were found in the program for preparing tape for wing templates, but the main portion of the program still has not operated successfully.

Six versions of a test cone with a sinusoidal cross-section, for which data was prepared on WWI, were machined recently with satisfactory results. The tape preparation program will be modified to produce data for a conic wing section.

- 142 D. A Study of Shock Waves has been undertaken in two dimensional solids subjected to impulsive loads. The analysis approximates the solid by a two dimensional grid with concentrated masses at nodal points. The response of this system is computed from a finite difference approximation to the differential equations of motion of this system.
:for Professor C.N.Norris, Department of Civil and Sanitary Eng.
:by E. Sydney (Res. Assist. CMMC), 30 hours
:DCL: WWI, 178 minutes

The program has been tested in its entirety and is operating satisfactorily. Approximately 10 hours of production runs are needed to complete this problem.

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- 147 C. Energy Bands in Crystals are being studied by finding solutions of the corresponding second order linear differential equations satisfying boundary conditions at the origin. The solutions are found approximately by using the Gauss-Jackson formula for forward integration. The solutions and their first derivatives are to be combined in a sum, the weighting factors being functions of an independent parameter.
 :for Professor J.C.Slater, Physics Department
 :by Dr. D.J.Howarth(DIC), 60 hours
 :DCL: WWI, 334 minutes

The first production run on the final program showed a logical error in the calculation of the matrix elements which has been corrected. Preliminary results after this correction indicate the convergence of the method to be better than expected; further investigation is underway.

The routine to solve secular equations involving non-diagonal overlap matrices, which forms part of the main routine of this problem, is being used to solve a total of 52 secular equations, of orders varying up to 20. This work has been undertaken for Dr. F. Herman, of R.C.A., Princeton. So far, 20 equations, mostly of low order, have been solved.

- 149 C. Digital Methods of Detecting Signal from Noise are being investigated. A sequence of binary numbers will simulate the message wherein regions of high density of ones are signal regions and those with low density of ones are noise regions. Various methods of detecting the change from one region to another, as well as the length and midpoint of the signal regions are being studied.
 :for J.V.Harrington, Lincoln Laboratory
 :by G.P.Dinneen(DDL)
 :DCL: WWI, 96 minutes

Several runs were made to determine the efficiency of the success run detector in locating regions of high density of ones in binary sequences of zeros and ones. The random process determined by the success run detector and the Bernoulli sequence is similar to a random walk problem with an upper barrier of 8, a lower barrier of 0, and reset conditions which are dependent on the position of the object. The probability of reaching the upper barrier for various density probabilities was determined.

- 159 B. Water Use In a Hydroelectric System. A hydroelectric system which has large water storage reservoirs has a certain flexibility of operation. Water can be stored during high flows and used during low. Just what use should be made of storage on any amount of water in the reservoirs, the probability distribution of future river flows, the anticipated demand for power, and the cost of obtaining power from sources other than hydro. A program is being written to solve the problem in some simplified cases. These cases have been patterned after the Columbia River system.
 :for J.D.C.Little (Res. Ass. Op. Res.)
 :by J.D.C.Little, 200 hours
 :DCL: WWI, 78 minutes

Since the last report was made on this problem, the following progress has been made:

1. A program is working which takes 2000 numbers, the historical records of river flow at Grand Coulee by weeks for 40 years, and puts them on the drum in a desired order.

2. A data processing program is working which takes these flows and deduces from them matrices of conditional flow probabilities. Computationally, this involves averaging into intervals of, say, months; taking 40 year sets of such flows for a given month; ordering them; forming then a smoothed cumulative frequency distribution; using this and a stored table to replace the actual flows with equivalent unit normal flows; correlating these with the corresponding unit normal flows of the preceding month; deriving a cumulative frequency distribution of the derivatives of the flows of one month from the mean implied by the preceding month's flow and the correlation between the months; deducing from this the conditional probability that the flow this month will be within certain limits, given that the flow the preceding month had a certain value.

- 161 C. Response of Mass-Plastic Spring System to Transient Loading: a 2nd order non-linear difference equation representing the response of building foundations to transient shock loads is being studied in order to develop criteria for the design of blast resistant foundations. The footing is represented by a concentrated mass, and the soil by a variable mass and an elastic-plastic spring. A Runge-Kutta fourth order integration procedure will be used.
 :for Professor R.V. Whitman, Asst. Prof. of Soil Mechanics
 :by S. Sydney (Res. Asst. CMMC), 30 hours
 :DCL: WWI, 22 minutes

Production runs are being made on the analyses of cohesive soils. These soils will be analyzed by the equations previously programmed for cohesionless soils to evaluate the strain rate effect.

- 166 C. Construction and Testing of a Delta Wing Flutter Model is being effected by replacing the actual wing by a structurally equivalent lattice network. An iterative procedure involving the evaluation of a matrix equation has been evolved for determining the bending and torsional stiffnesses of the component members of the network.
 :for M. M. Chen (DIC)
 :by S. Gravitz (Res. Asst. Aero. Eng.), 55 hours
 :DCL Staff: Demurjian, .25 hour; Denman, 3 hours; Porter, 7.5 hours; WWI, 152 minutes

Testing is being continued on the first cycle of the iteration procedure. Numerous difficulties have been encountered and not all of them have as yet been resolved.

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- 167 D. Transient Effects In Distillation are being calculated by solving sets of simultaneous non-linear ordinary differential equations using numerical integration methods. Several of the most important types are being explored and results of specific cases will be correlated.
 :for Professor E.R.Gilliland, Chemical Engineering Department
 :by J.F.O'Donnell(Res. Asst. CMMC), 40 hours; Myers, 40 hours;
 Polk, 40 hours; Smith, 40 hours
 :DCL Staff: Denman, 1 hour; Porter, 2 hours; WWI, 285 minutes

The machine time used during this period was divided almost evenly among the problems of transients in continuous distillation, tune-up in batch distillation, and take-off in batch distillation.

The program for tune-up in batch distillation is now operating successfully. Part of the results are obtained on the scope, the rest on magnetic tape for delayed print-out. Several cases have been run. Runs will be continued.

Myers

The programs for transients in continuous distillation have been used to obtain more data. Correlation of the results is being attempted now.

Polk and Smith

The program for take-off in batch distillation has been used to obtain more exploratory data. Some attempts at correlation have been unsuccessful, but there are still a number of methods yet to be tried.

O'Donnell

- 169 B. Utilizing a General-Purpose Digital Computer in Switching-Circuit Designing to find the most economical physical realization of a prescribed switching function. The procedure chosen is one that will yield a minimum sum-of-products form of any given transmission function within certain limits (i.e., no more than 15 variables or 2^9 terms at present).
 :for Professor D.A.Huffman, Electrical Engineering Department
 :by E.C.Hoy (Res. Assist. E.E.), 60 hours
 :DCL: WWI, 16 minutes

The program for the last rule to be applied in the simplification procedure, $XY + YZ + ZX' = XY + ZX'$ (not $XY + ZY'$), has been successfully run and has been attached to the rest of the program, completing an acceptable solution to the original problem. All of the timing runs have not yet been made so the results are not completely analyzed.

- 171 C. Improved Power Spectrum Estimates are to be obtained by investigating modifications to existing techniques for the numerical calculation of Fourier cosine transforms to minimize the effects of truncation and to supply a confidence curve, based upon the interval variations of the calculations, which will help to evaluate the significance of the resulting spectrum estimate. The object is to obtain a method which will give significantly better power spectrum estimates than can be obtained using existing techniques.

:for J.E.Ward, Project Engineer, Servomechanisms Laboratory

:by D.T.Ross (DIC), 40 hours; Hamilton, 20 hours

:DCL: WWI, 387 minutes

The new methods for improving frequency resolution were partially tested in three runs with various modified weighting functions. Results were satisfactory and the methods will be perfected some time in the future. A long run using an empirical autocorrelation function of 500 points was finally run successfully. It was found that the provision in the new PA routines for stopping after an isp, icp, or ict instruction was the cause for record alarms on si 300 (which selects the MIR). Once found, the difficulty was easily corrected. The runs to be used in the SM Thesis report are now complete and problem 171 will be discontinued temporarily, to be opened again at a later date to complete the resolution studies.

- 172 B. Overlap Integrals of Molecular and Crystal Physics. Two-center overlap integrals are to be evaluated between various Slater atomic orbitals, which are of the form: (power of r) \times ($\exp -ar$) \times (spherical harmonic). By use of prolate spheroidal coordinates, formulas in terms of simple functions can be derived for these integrals but are unsatisfactory because they are of formidable complexity and have false singularities. The integrals are evaluated by recasting the expressions in terms of spherical Bessel functions of imaginary argument, which can be generated by a high-speed computer.

:for Professor J.C. Slater, Physics Department

:by F.J. Corbato (Res. Asst., CMMC), 50 hours

:DCL: WWI, 195 minutes

With one minor exception, all the control routines have been tested. In addition, the $1s$, $2s$, and $2p$ wave functions of the Carbon $3p$ configuration as well as the $2p$ function, have been fitted to three degrees of accuracy by means of the curve-fitting routine. It is planned to make an over-all test of the problem by doing a Tight-Binding calculation of the Graphite and Diamond crystal structures.

173. MIT Course 6.537 Spring Term 1954. Twelve students have enrolled in the Electrical Engineering course 6.537, entitled Digital Computer Applications Practice, being given by Professor C.W. Adams. The purpose of the course is to study the advanced preparation of coded programs for automatic, electronic digital computers, in particular, for Whirlwind I. This study will include techniques for handling storage and terminal equipment, detecting errors and mistakes in programs,

the control of scale factors, and the use of subroutines. Each student will program, prepare on punched tape, and execute on Whirlwind, one problem of his own choosing, making use of the CS II computer.

:DCL Staff: Combelic, 6 hours; Denman, 2.5 hours; WWI 180 min.
Kopley, 2 hours

173-100 This problem is concerned with the simultaneous solution of a set of linear algebraic equations by the Crout method (see last bi-weekly) This has been accomplished on a 4th order set. A 6th order set is now being attempted.

Arbuckle

173-101 This problem is concerned with the computation of a double integral of a probability equation. (See last bi-weekly for more details). The last set of tapes run on the machine attempted to determine the representation the computer makes of the numbers that it computes. This was done in an attempt to attain 7th and possibly 8th place accuracy in the results. Accuracy at present is good to 7 places.

Brun

173-102 The complete program for the solution of the delay line spurious noise problem (described in bi-weekly of April 18) has been prepared. It was necessary to make use of the auxiliary drum for temporary storage because of the length of the program. Difficulty was encountered in determining how to use this facility in connection with CS II conversion. Shortly, the program will be run with additions to print out intermediate results to check the computation.

Dennis

173-103 This problem involves taking discrete data over one period and forming the Fourier components for this data. (described in bi-weekly of April 18) Successful results have been obtained.

Eccles

173-105 This problem may be described as follows: given arrival times at more than two locations of a shock from an earthquake, to find the time of occurrence and the location of the quake. This is done by a least square fit of empirical time - distance curves.

The program falls into three nearly equal parts. Part I determines the distance between two points on a sphere. Part II makes the least square fit. Part III solves a set of three linear algebraic equations.

All three parts are written and part III was run successfully. Part I now works on some numbers. Parts I and II are now being tested with the hope that they will be run successfully in the near future.

Grine

173-106 This problem is concerned with finding the roots of a polynomial by Newton's method. The problem has been coded and run successfully on a limited amount of data .

Jones

173-107 This problem concerns the solution of a set of linear equations by the Gauss-Jordan reduction method (described in the May 2, 1954 Biweekly). As of this writing, incorrect results have been obtained. The routine is fully developed (in absolute address form) except for the detection of the error. The problem has been rewritten in floating address form. By the end of the term, it is desired to incorporate a test for a singular set of equations.

Kreide

173-110 This problem seeks to produce an arc tangent routine. The original routine was found to work but left much to be desired, timewise. Upon the suggestion of Dr. Denman of the DCL Staff, a Rand approximation was coded and found to be not only much faster, but shorter as well. It was hoped that the coefficients for a higher ordered approximation were close enough to those appropriate for a lower approximation that one routine could be used for varying accuracy needs. It was found that the coefficients were not nearly enough alike to permit this.

The remaining problem is to investigate whether or not the result of using an eight-term approximation produces an error curve which is good enough to justify its use over that of a six term approximation. This study will make use of manual conversion from octal to decimal to ensure accuracy in the eighth decimal place.

Petrick

176 B. Connector Provision in Automatic Telephone Exchanges.

The Traffic Problem - a particular solution designed to meet conditions encountered by Line Finders and Connectors in automatic step-by-step telephone exchanges. The treatment allows for interaction of sources and sinks - that is, a terminal may serve either as a source or as a sink, but not as both simultaneously. Consideration is also given to the effect of a distribution of the strengths of sources (or sinks) rather than assuming that all terminals are of equal strength.

:for Professor C. Tucker, Elect. Eng. Dept.

:by B. Marrows (Grad. Student)

:DCL Staff: Porter, 8 hours; WWI, 371 minutes

The general traffic problem consists of determining the relationship between A, the amount of traffic offered to a number of channels, N, the number of channels available, and B, the probability of an item of traffic failing to secure a channel because of the limited number of channels available.

The traffic considered may be telephone calls, customers at a store, automobiles, visitors at an exhibition, telegrams to be sent over a communications system, etc. The corresponding channels might be switching circuits, shop attendants, lanes in a highway, turnstiles at an entrance, or bandwidths of the system.

Many solutions to the problem have been obtained under varying sets of assumptions, but none is satisfactory for the case of connectors in step-by-step automatic telephone exchanges. All solutions available tend to over-estimate requirements by amounts varying between 5% and 20%. The most accurate traffic tables available are based on the binomial formula and were first applied in 1905 by E.C. Molina.

Due to the rapid development in electronics, and hence in toll service, most of the recent work on telephone traffic theories has been devoted to the problem of lost calls delayed. However, the connector circuit is still a vital link in automatic telephone connection and inexpensive as it may be by comparison with the toll circuit, its importance lies in the large number of connector groups in operation. (There are approximately a quarter of a million groups in use in the United States alone, with each circuit costing approximately \$100.)

It is believed that one of the main reasons for the over-estimation of connector circuits, as indicated by existing traffic tables, is that none of the theories on which they are based makes allowance for sources also being available as sinks. So far as can be ascertained, no endeavor has been made to allow for this inter-action of sources and sinks. Consideration of the inter-action of sources and sinks produces the following equations.

$$k P_k = \frac{P}{1-P} P_{k-1} \sum_{w=0}^{s-k} (S-w-k+1) Q_w \quad (1a)$$

(k=1,2,3,...,S)

$$\sum_{w=0}^S P_w = 1 \quad (1b)$$

where P_w is the probability that exactly w circuits will be simultaneously engaged as sinks.

S is the number of terminals which may serve either as a source or sink.

P is the probability of any given terminal being in use as a source at any given instant, (assumed constant for all terminals).

Q_w is the probability that exactly w circuits will be simultaneously engaged as sources, (assumed equal to P_w).

From solutions to the above equations it is then required to find the relationship between $A=PS$ and N for a given B in the following equation.

$$B = \frac{1}{S(1-p)} \sum_{k=N}^{S-1} \sum_{w=0}^{S-k-1} (S-k-w) Q_w P_k \quad (2)$$

A simple iterative procedure for the solution of these equations, which converges rapidly in the cases taken as samples, has been devised, and the problem is highly suited to electronic computation over the following ranges of values.

Equation (1) $p = 0.02$ (0.01) 0.20
 $S = 25$ (25) 200

Equation (2) $N = S_p, S_{p+1}, S_{p+2}, \dots$ until $B \leq 0.001$.

Only basic procedures (addition and multiplication) are required on an iterative basis. Reasonable first approximations are available so that it is expected that on the average four or five iterations will be necessary to meet the required accuracy. Solutions to equation (1) are required to four decimal places while three decimal places will be sufficient for solutions to equation (2).

After checking the results of the above work with observed statistics of telephone traffic, it is proposed to introduce the further refinement of determining a "reasonable" estimate of the distribution of p within a group of S terminals, and endeavoring to determine new equations to replace (1) and (2) above on the basis of the additional assumption. If these equations can be obtained, they will certainly be more complicated than (1) and (2) above, and again the services of the electronic computer will be required.

Although the purpose of the paper is to solve a particular problem in telephone traffic engineering, and parameters have been selected accordingly, results will also be applicable to other fields where conditions approximate those set out in the assumption.

A simple iterative procedure in which each P_k was calculated in turn as a function of the best available approximation of all the other P_k was first utilized. Although the method converged for small values of P , it diverged for large values of P .

A second procedure in which equations (1a) and (1b) were reduced to a set of linear simultaneous equations was then devised. These equations were then solved by conventional means, and iteration of the process was found to converge over all values of p required.

Using the second method, results were obtained for $S = 25$, and $S = 50$, over the required range of p . Results indicate that existing traffic tables overestimate requirements by values varying from 5% at $p = 0.02$ to 22% at $p = 0.20$ for $S = 25$ and from 4% at $p = 0.02$ to 20% at $p = 0.20$ for $S = 50$.

Since the time of calculation increases approximately as the square of S , it was decided to calculate results for $S = 100$ and $S = 200$, and interpolate for intermediate values of S as required.

In a further endeavor to reduce computer time, only selected values of p were used, with the idea of estimating intermediate values from a curve of results. On this basis results have been obtained for 6 values of p with $S = 100$, but results from further values of p will be necessary before reasonably continuous results can be obtained.

- 180 B. Crosscorrelation of Blast Furnace Input-Output Data. It is desired to evaluate a system function relating iron output to coke input. This will be done by performing a cross-correlation of the quantities and then evaluating the Fourier transform of the crosscorrelation function.
 :for Professor D.P.Campbell and C.A.Powel, Electrical Eng. Dept.
 :by R.G.Mills, 25 hours; H.J.Scholz, 25 hours
 :DCL: WWI, 143 minutes

The final computation of the crosscorrelation function for single-furnace data was completed, and the sine portion of the Summer Session sine-cosine subroutine was finally optimized. A minor alteration in the cosine routine remains to be tested.

The remainder of the computer time was employed in error-diagnosis of a modification of a Fourier Transform program written by D.T.Ross in connection with Problem #171; this work is still in progress, and when finished, will complete the problem.

- 182 C. Crystal Structures. In solving a crystal structure, the corrections to the atomic parameters are obtained by a least squares method, as a set of n simultaneous linear equations in n unknowns. n may be 30 or more. It is required to solve this determinant.
 :for Dr. S.C.Abrahams, Laboratory for Insulation Research
 :DCL Staff: Arden, 4 hours; WWI, 111 minutes

This problem makes use of the matrix diagonalization procedure developed by Dr. A. Meckler under problem #134.

The inversion of the 30th order matrix associated with this problem is proving difficult because of the high order of the matrix. During the past biweekly period one long run was carried out and stopped manually after 111 minutes. The inverse at this point is estimated to be accurate to approximately one significant figure. Convergence is slow because of the large number of drum references and the fact that the program is written almost entirely in the interpretive code. The results

indicate that for matrices of approximately this order, the program should be rewritten minimizing use of the interpretive code and drum references.

183 D. Blast Response of Aircraft. The present program is the first phase of a study attempting to establish a lethal criterion for airplane structures upon encountering a blast gust. For the purpose of examining some of the simplifying assumptions necessarily made in actual aircraft structures and of investigating the feasibility of a step-by-step solution in solving a nonlinear dynamic problem, this first phase treats the simple case of a cantilever, uniform, weightless beam subject to a triangular pulse at its tip mass. The structural characteristics of the beam are assumed to be such that its moment-curvature curve is bilinear.

:for H. Lin, Aeroelastic Laboratory

:by D. Sternlight, 40 hours

:DCL: WWI, 111 minutes

All quantities in the governing nonlinear equations have been nondimensionalized. The parameters of each case to be computed by WWI are

K = buckled slope of the moment-curvature curve

ϵ_y = yield curvature of the beam

τ_f = duration of triangular pulse

The ultimate quantities desired from the present program are

ϵ_a = maximum curvature at the root

$R' = \frac{\epsilon_a}{\epsilon_y}$ = ratio of maximum curvature to yield curvature

$\tau_a \Delta \tau$ = time to reach maximum root curvature.

For a given beam of known ductility factor defined as

$$R = \frac{\epsilon_c}{\epsilon_y}$$

where ϵ_c = failure curvature of the beam, the calculated value of R' will show that the beam will fail if

$$R' \geq R$$

A parameteric-variation of K , ϵ_y and τ_f will result in a plot of a family of curves from which a lethal criterion for this beam may be obtained under the aforementioned forcing function.

The numerical procedure employed in the step-by-step solution is the classical approach used in solving nonlinear differential equations of second order. Briefly it can be outlined as follows:

1. The velocity of the tip mass at any instant is extrapolated from the accelerations at previous 3 instants.
2. The displacement of the tip mass is then integrated from the velocity obtained above by Simpson's Rule.
3. The net tip force necessary to produce this tip displacement is then solved by the Newton-Raphson iteration method from the cubic structural equation.
4. The acceleration of the tip mass at this instant is then determined from the equilibrium equation.
5. The root curvature at this instant is also calculated.
6. The cycle is repeated, until a maximum value of root curvature is obtained.
7. To start the recurrent procedure outlined above, the necessary quantities at the initial 3 instants are determined by a different method similar to NACA TN 2060.

The formulation of the simplified case was programmed. At present the program is being corrected for the plastic range. The linear range has been run with results in agreement with actual analytic solutions to three significant figures.

184 D. Scattering Electrons from Hydrogen. A stationary expression has been formed for the scattering amplitude. Parts of this expression have been reduced (for a given trial function) to 2 dimensional integrals which are to be solved on WWI by Simpson's Rule.

:for Professor P.M.Morse, Physics Department
:by M.C.Newstein (Res. Assist), 20 hours
:DCL: WWI, 31 minutes

This problem originated in a suggestion by Professor P.M.Morse that recently developed mathematical variational techniques, together with machine methods of computation, could be used to solve certain heretofore incompletely treated problem in atomic scattering. In particular, the low energy scattering of electrons from hydrogen, taking into account excitation and exchange falls into that category. Part of this problem, which forms M.C.Newstein's PhD thesis, has been to construct a stationary expression for the scattering amplitude, and to study the multidimensional integrals that are involved. These have been reduced to two dimensional integrals which may be solved to the required accuracy on WWI for certain trial functions. If the calculation is successful, it is expected that the method will prove important in the calculation of the cross-sections for scattering from more complicated atomic and nuclear systems.

An expression for the differential amplitude for the scattering of electrons from hydrogen atoms has been devised, which has the property

of being stationary with respect to variations in the wave function. The integrals in this expression have been reduced, for a given trial wave function, to a two-dimensional form. These two-dimensional integrals are to be evaluated by Simpson's Rule for various values of angle of scattering and incident electron energy.

At present a program to construct the integrand is being written.

1.3 Operating Statistics

1.31 Computer Time

The following indicates the distribution of WWI time allocated to the S&EC Group.

Programs	66 hours, 26 minutes
Conversion	12 hours, 58 minutes
Magnetic Drum Test	41 minutes
Magnetic Tape Test	1 hour , 09 minutes
Scope Calibration	38 minutes
Demonstrations(#131)	<u>5 hours, 27 minutes</u>
Total Time Used	87 hours, 19 minutes
Total Time Assigned	90 hours, 04 minutes
Usable Time, Percentage	96.9%
Number of Programs	378

1.32 Program Time Distribution

The following table attempts to show how the WWI time expended on S&EC programs was distributed with respect to machine runs that gave meaningful results (productive computer time) and runs that gave unsatisfactory results (lost computer time). Productive computer time is subdivided to indicate the time involved in actual computations as contrasted with the time expended getting information out of WWI. Computer time lost is subdivided to show the portion of time lost due to errors in the programmer's formulation of his problem (logical errors); due to errors in the programmer's use of the WWI code, CS Conventions, etc. (technical errors); due to tape preparation errors; due to errors by the S&EC computer operators in running the program; due to malfunctioning of terminal equipment; and finally due to miscellaneous causes.

These times are determined as percentages of the time listed above in section 1.31 for programs. The times used in computing these figures are extracted from the biweekly report forms submitted by the various programmers who have used S&EC allocated WWI time.

1. Productive Computer Time
 - Computation 55.2%
 - Output 18.3%
2. Computer Time Lost Due to Programmers' Errors
 - Technical 18.7%
 - Logical 2.4%
3. Computer Time Lost Due to Other Difficulties
 - Tape Preparation 1.8%
 - Operators' Errors 0%
 - Terminal Equipment Malfunction 1.7%
 - Miscellaneous 1.9%

1.33 Tape Preparation (M. Mackey)

An attempt is being made to obtain some idea of the time expended in the preparation of tapes. During the past biweekly period a check was made on the tapes processed.

Due to the variations in procedures involved we have distinguished among original complete tapes and the following three types: typed modifications - changes of 11 or more registers which must be typed, converted, then attached to the main program or changes which must be made in the body of a Flexowriter tape; manual modifications - changes punched directly in 556 form and attached to a converted tape; combined tapes - which require duplication of two or more complete tapes.

The following information was compiled:

	<u>Complete Tapes</u>	<u>Typed Mods</u>	<u>Manual Mods</u>	<u>Combined Tapes</u>
No. of Tapes	111	60	29	6
No. of Registers	23114	1335	158	
Time Consumed	68hrs.3min.	35hrs.4min.	2hrs.48min.	3hrs.44min.

Thus, it may be seen that the average length of an original complete tape is 208.2 registers requiring 36.8 minutes to prepare. A typed modification averages 33.4 registers in length and required 52.6 minutes to prepare while Manual Modifications average 5.4 registers and require 5.8 minutes for preparation.

In addition to the time accounted for above, a total of 5 hours and 15 minutes was spent printing computer results for photographic reproduction.

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2. COMPUTER ENGINEERING

(S.H.Dodd)

Computer operation has been good during the past biweekly period. The applications group estimates that about 95 percent of assigned computer time was usable.

Marginal-checking procedures have been extended to the intervention registers and the auxiliary-storage sections of the buffer drum. Reliability of the terminal equipment has been increasing regularly as preventive-maintenance routines have become more comprehensive.

The major portion of the bay wiring for the electronic-write-selection system on the auxiliary drum has been finished. A complete changeover to electronic switching may be delayed by a strike currently in progress at the plant which is our source of writing transformers.

2.1 WWI System Operation (A.J.Roberts, L.L.Holmes)

Computer operation was interrupted for approximately 8 hours as a result of a gassy 6SL7 in a digit-plane-driver. The trouble was first noted following a failure of the -450-v bias supply. The disturbance at first was of an intermittent nature but eventually became steady state. The replacement of the plug-in unit containing this tube corrected the trouble and permitted the resumption of normal operation.

An additional hour of computer time was lost because of drum parity alarms caused by close timing at digit 15 of the parity register. The trouble occurred only during the blackout of test storage. The difficulty was corrected by introducing an additional 0.1 μ sec. delay of the parity count pulse. This enabled the flip-flop to become stable before sensing its gate tubes.

In the previous biweekly report we mentioned that an investigation of approximately 200 allied plug-in relays revealed that 8 had unsoldered connections. A check of 1140 others has been made since then. As a result, 69 additional relays were discovered with faulty connections. We have approximately 500 more relays to check on our next installation day. Anyone desiring additional information concerning these relays may obtain it by referring to the Component Report for Job #001-012 made by B. B. Paine.

2.11 Magnetic Tape (A.X.Perry)

Margins of the read/record circuits have been improved with the replacement of a few drifting components during this period.

The delayed print-out equipment for Unit 2 is now complete, but unfortunately we have been plagued with a timing difficulty which is more critical during the punch-out operation. More time will be utilized toward the tracking down of this fault.

Unit 2 and Unit 3, IOS selection, can now be interchanged by the manual switching of the Mod. III transfer panel, and the FL printers are easily changed to operate from either unit by the Mod. II transfer panel.

The FL machine normally tied into Unit 2 can also be switched to operate as a manual typewriter for paper-tape preparations. A memo will be proposed shortly in regard to the flexibility of operations available with two complete delayed outputs.

2.2 Terminal Equipment (R.H.Gould)

The changes to block control were accomplished very successfully.

It is planned to provide an indication when the Fairchild Display Scope camera runs low on film to keep the computer from attempting to take pictures without film. As now planned, contacts in the camera magazine will cause the si 0004 order to "switch to push-button" as well as index the camera. Enough film will be in the magazine at this time to provide a convenient leader for processing. It will not be possible to suppress this "Sw to PB" action, but if it is desired to take only a few more pictures on the same film the "restart" button will start the program, and the same action will occur on the next si 0004 order. The green alarm light will flash, and the alarm gong will ring as with the si 0000 and si 0001 orders, but 0004 will be in the in-out switch to provide a unique indication.

2.21 Marginal Checking (T.J.Sandy)

The criterion for assigning variable voltage lines to the in-out element has been completed, and that part of the in-out element in the WWI computer room has been divided according to this criterion.

The necessary WWI modifications for the reassigning of variable voltage lines will be started during the coming biweekly period.

2.22 Magnetic Drums (H. L. Ziegler)

Digits 8-15 of the new writing circuits in the auxiliary drum have been thoroughly checked out and are ready to be made a permanent part of the drum system. These eight digits will be put into service as rapidly as installation periods permit.

Strike troubles are holding up the transformers necessary to complete the remainder of the Type 3 chassis now in the construction shop. Delivery of these completed chassis will determine the time for installation of digits 0-7 of the new auxiliary-drum writing circuitry.

(L.D.Healy)

A checking procedure was devised for locating failures which cause persistent parity alarms in the auxiliary-drum system.

Work was continued on a technician's handbook covering the drum systems.

2.23 Intervention Registers (F.E.Irish)

The marginal-checking procedure for the intervention registers has been improved to the point where any one of the 1500 crystals used in the register can be "pinpointed" if its back resistance falls below 50,000 ohms.

2.24 Ferranti PETR (F.E.Irish)

The installation of the production model of the Ferranti PETR has been completed. From the standpoint of the computer operation the only difference will be that the reader can now be stopped at any time by pressing the "stop" button and then restarted by using the "restart" button.

3. ADMINISTRATION AND PERSONNEL

New Staff (J.C.Proctor)

Lloyd B. Smith is working as a DDL Staff Member and has been assigned to the Administration and Service Group. Until recently, he has been working as a Design Engineer at the Laboratory for Insulation Research.

Staff Termination

Robert Maglio

New Non-Staff (R.A.Osborne)

Gerald Avarbock is a new technician in the Construction Shop.

Donald Duncklee has also joined the Construction Shop as a technician.

Ruth Hobbs is Steve Dodd's new secretary.

Terminated Non-Staff

Renee Feinstein

Carol Small

Marilyn Susskind

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4. LIBRARY ACCESSIONS LIST

The following material has been received in the Library, Barta 109.

Library Files

<u>No.</u>	<u>Author or Source</u>	<u>Title</u>
C14	Ballistic Research Lab.	Automatic Error Control, The Initial Value Problem in Ordinary Differential Equations - Report 893
C134	Allen Keller and R. A. Butterworth	Programming for the IBM 701 Electronic Data Processing Machine with Repetitively Used Functions
C135	Remington Rand	The A-2 Compiler System Operation Manual
C136	Roy Goldfinger	New York University Compiler System
C137	Bailey T. Wade (Ballistic Research Lab.)	Machine Solution for a System of Linear Algebraic Equations
C138	Wolfgang Wasow	On Small Disturbances of Plane Couette Flow - Reprint
C139	Wolfgang Wasow	Asymptotic Solution of the Differential Equation of Hydrodynamic Stability in a Domain Containing a Transition Point - Reprint
C140	James E. Robinson (U. of Ill. Internal Rept.)	Error Detection and Correction in Binary Parallel Digital Computers

The following material is available from the Document Room, W3-141, x3489.

M-2704	A. Falcione	Distribution Procedure for Laboratory Memoranda
M-2715	S. Pest	Programming for the Ferranti Photoelectric Tape Reader
M-2728	G. Young	Increased Facilities for Visual Display in the WWI Input-Output
M-2729	G. Young	Paper Tape Units and Printers in the WWI Input-Output System

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The material listed below can be obtained from the Barta 006 Library, x3792.

<u>No.</u>	<u>Source</u>	<u>Title</u>
2694	BRL	An Ordvac Floating Binary System
2697		Ms of ACM Speech, Sept. 1953 Meeting by J. Runyon
2698	BRL	The Reflection of Plane Electromagnetic Waves at a Plane Boundary of a Conducting Medium
2700	BRL	The Solution by Iteration of Nonlinear Integral Equations
2701	Nav.Res.Lab.	Measurement of a Time Varying Complex Propagation Constant in the Microwave Region
2712	BSTJ/Repr.	Error Detecting and Error Correcting Codes
2713	Bell Lab.	Switching Systems as Mechanized Brains
2714	Electronic Eng.	Mechanized Reasoning - Logical Computers and their Design
2716	NBS	Some General Theorems on Iterants
2719	IRE	IRE Professional Group on Communications Systems Newsletters
2722	MIT	Mechanical Translation - Bibliography
2724	Fortune Mag.	The Information Theory
2727	IRE	IRE PG on Electronic Computers Newsletter
2730	NBS	The Diode Capacitor Memory Report
2734	IRE	Proceedings of the Eastern Joint Computer Conference, AIEE, IRE, ACM, Washington, D.C.
2742	NBS	System Design of the SEAC and DYSEAC
2743	NBS	System Organization of the DYSEAC
2744	BRL	Ordnance Computer Newsletter
2746	DGL	Notes on Digital Computers and their Applications Summer Session 1953
B-279	Prentice-Hall	Information Theory
B-280	Prentice-Hall	Principles of Automatic Controls
B-282	NBS	Simultaneous Linear Equations and the Determination of Eigenvalues
B-283	Prentice-Hall	Differential Equations in Engineering Problems
B-285	Buttersworth	Automatic Digital Calculators

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