A U T O N E T I C S

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TITLE: **Improved Method for Lagrangian Interpolation**

PURPOSE: To indicate a simplified method of writing Lagrange's Interpolation Formula

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CONTENTS: 1. INTRODUCTION

Lagragian Interpolation lends itself nicely to machine calculation, but when the indices are equal the missing terms of the products require special treatment. The following approach decreases the number of multiplications and divisions required and practically eliminates the necessity of checking for index equality.

2. METHOD

If we define

$$P_k(X) = \frac{(X-X_1)(X-X_2)\ldots\ldots(X-X_k)\ldots\ldots(X-X_{n-1})(X-X_n)}{(X-X_k)} = \prod_{\substack{i=1 \\ i\neq k}}^{n} (X-X_i),$$

then Lagrange's interpolation formula [1] becomes

(1) $$y = \sum_{k=1}^{n} \frac{y_k P_k(X)}{P_k(X_k)}$$

Assume $X \neq X_k$ for all $k = 1, 2, \ldots, n$. (if $X=X_k$, $y=y_k$).

We may then write

$$y = \sum_{k=1}^{n} \frac{y_k \prod_{i=1}^{n} (X-X_i)}{(X-X_k)\, P_k\, (X_k)} \quad \text{or,}$$

$$y = \prod_{i=1}^{n} (X-X_i) \left[\sum_{k=1}^{n} \frac{y_k}{(X-X_k) P_k\, (X_k)} \right] \quad \text{or,}$$

(2) $$y = -\prod_{i=1}^{n} (X-X_i) \left[\sum_{k=1}^{n} \frac{y_k}{(X_k-X)\, P_k\, (X_k)} \right]$$

Write

$P_k\, (X_k) = (X_k-X_1)\, (X_k-X_2) \ldots (X_k-X_{k-1})\, (X_k-X_{k+1}) \ldots (X_k-X_{n-1})$

(X_k-X_n).

The denominator term (X_k-X) of (2) is similar to the terms of $P_k\, (X_k)$, so let

$P'_k\, (X_k) = (X_k-X)\, P_k\, (X_k) =$

$(X_k-X_1)\, (X_k-X_2) \ldots (X_k-X_{k-1})\, (X_k-X)\, (X_k-X_{k+1}) \ldots (X_k-X_{n-1})\, (X_k-X_n)$.

(Since X_k is needed as a constant minuend in forming $P'_k\, (X_k)$ it can be replaced in the table by X when it is picked up, and restored after $P'_k\, (X_k)$ is formed).

We now have

$$y = -\prod_{i=1}^{n} (X-X_i) \left[\sum_{k=1}^{n} \frac{y_k}{P_k'(X_k)} \right]$$

Assuming that the products are set originally to -1 for $\prod_i$ and +1 for the P_k', this method requires $(n+1)^2$ or n^2+2n+1 multiplications and divisions, as opposed to $2n^2$ for the conventional method. It also simplifies the "housekeeping" by eliminating the tests for i=k.

(It is, of course, nicest if there exists an instruction to exchange the contents of the arithmetic register with the contents of a word in memory).

REFERENCE

1. Ivan S. and Elizabeth S. Sokolnikoff : "HIGHER MATHEMATICS for ENGINEERS and PHYSICISTS," McGraw-Hill Book Co., Inc., New York, 1941, 553.

INFORMATION TO: All Concerned

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