RECOMP II USERS' PROGRAM NO. 1102

PROGRAM TITLE: 125-R FIRST ORDER SYSTEM (FLOATING POINT)

PROGRAM CLASSIFICATION: Subroutine

AUTHOR: W. Wellman
Servomechanisms, Inc.
Research Division
Goleta, California

PURPOSE: To solve the equation,

$$c - r = \tau(dr/dt)$$

for r, where c is given as a function of time.

DATE: 11 May 1961

Published by

RECOMP Users' Library

at

AUTONETICS INDUSTRIAL PRODUCTS
A DIVISION OF NORTH AMERICAN AVIATION, INC.
3400 E. 70th Street, Long Beach 5, Calif.

FIRST ORDER SYSTEM

(floating point)

PURPOSE:

To solve the equation,

$$c - r = \tau (dr/dt)$$

for r, where c is given as a function of time.

METHOD:

The following recursion relation is used:

$$r_n = A(c_n - c_{n-1}) + B(r_{n-1} - c_{n-1}) + c_n$$

where: $A = \frac{\tau}{\Delta t}(1 - \epsilon^{-\Delta t/\tau})$, $B = \epsilon^{-\Delta t/\tau}$

giving the value of r at the n'th instant in terms of c at the n'th instant and the previous r and c at the (n-1)'th instant. Since all storages are upgraded internally, it is only necessary to enter the routine with successive c's, exiting with the successive r's, if the interval between n and n-1 is unchanged. The routine assumes that c varies linearly with time between points, so that it is only necessary to choose the points on c so that c is sufficiently well represented as a series of straight lines.

RESTRICTIONS:

$\tau > 0$

CALLING SEQUENCE:

TRA (location of this routine); exit is to next half-word. On entry, A and R should contain c_n; on exit, A and R will contain r_n.

PARAMETER STORAGE:

The following locations must be filled as indicated with the correct initial conditions before this subroutine is used. The r and c values are upgraded automatically; time is assumed to increase by an amount Δt each time through the routine.

Lo + 30 r	Lo + 34 Δt
Lo + 32 c	Lo + 36 τ

In addition, the following are calculated and made available in the locations shown, but need not be filled with initial conditions.

Lo + 42 c-r	Lo + 44 dr/dt

New r_{n-1} or c_{n-1} values can be stored in the indicated locations at any time, resulting in a discontinuity; thus discontinuities in either r or c can be solved for accurately.

Δt may be changed as desired to provide larger or smaller steps in time, to show more or less detail, or to suit the nature of the input function c. τ may also be changed in a step-wise fashion, thus yielding an approximate solution to many non-linear equations.

ROUTINE REQUIRED:

Exponential (AN-044) in 1050; called for by TRA instruction in (Location of this routine + 46)

STORAGE REQUIRED:

$100_{(8)}$ sectors, plus use of both high speed loops.

REMARKS:

The solution is "exact" in that the recursion relationship will give the exact solution (except for round-off errors) if c varies linearly between points and τ is a constant. Non-linear and time-varying solutions can be approximated by choosing Δt small enough so that τ is essentially constant over each computation interval. The constants A and B in the recursion relationships are computed with an accuracy within 0.5×10^{-10}.

TIME:

0.8 seconds to solve for r_n and evaluate A and B.

0.2 seconds to solve for r_n (no change in Δt or τ).

CHECKOUT:

This routine generated correct solutions to:

(1) simple step response: $r = \frac{1}{s(s+1)} = 1 - \epsilon^{-t/\tau}$

(2) non-linear system step response: $r = \frac{1}{s(1+s+ts)} = \frac{t}{1+t}$

FILE UNITERMS:

First Order System, Differential Equation, Time Constant

SM/I, Research Division, Digital Computing Facility

Program No. 125 R Title FIRST ORDER SYSTEM

Programmed by: Date 5-11-61

Loc'n	Cm'd	Addr.	Contents	Accumulator	b	Remarks
00	F.S.T	440				
	C.T.V	200				
1	S.A.X	77640				
	T.R.A	77720	→ 22			
2	F.C.A	400	e^{-x}			← here if $x \geq \frac{1}{8}$
	F.S.B	560	1			
3	F.D.V	200	$-x$	$(1/x)(1-e^{-x})$		
	T.R.A	750				
4	+.40	00000	$A = \frac{1}{x}(1-e^{-x})$			
	-.00	00000				
5	+.00	00000				
	-.00	00001				
6	+.00	00000	43 @ 38			
	-.00	00530				
7	F.C.A	560	1			← here if $x < 2^{-40}$
	T.R.A	750				
10	F.C.A	440		c		
	F.S.T	320	c_0			
1	F.S.B	77620		$c - c_0$		Recursor
	F.M.P	040		$A(c-c_0)$		
2	F.S.T	77700	Temp			
	F.C.S	77600		$-r_0$		
3	F.A.D	77620	c_0	$c_0 - r_0$		
	F.M.P	400	B	$B(c_0 - r_0)$		
4	F.A.D	77700		$A(c-c_0)-B(r_0-c_0)$		
	F.S.T	420		$\varepsilon = c - r$		
5	F.D.V	77660	τ	$\dot{r}$		
	F.S.T	440				
6	F.C.A	320		c		
	F.S.B	420	$c - r$	r		
7	F.S.T	300	r_0			
	T.R.A	~~00000~~	EXIT	r		
20	+.00	00000	$-\Delta t/\tau$			
	-.00	00000				
1	+.00	00000				
	-.00	00000				
2	A.D.D	77770				
	C.T.L	300	L_1			
3	S.T.O	170	exit			
	F.C.S	77640	$-\Delta t$			
4	F.D.V	77660	τ	$-\Delta t/\tau$		
	F.S.T	77640				
5	F.S.B	77700	old $(\Delta t/\tau)$			
	T.Z.E	760	→ no change			
6	F.C.A	77640				
	T.R.A	460				
7	F.S.T	300	bump			
	T.R.A	00001				

Program No. 125-R Title

Programmed by: Date 5-11-61

Loc'n	Cm'd	Addr.	Contents	Accumulator	b	Remarks
30			r_0			"INITIAL CONDITION."
1						
2			c_0			
3						
4			Δt			
5						
6			τ			
7						
40	+40	000.00	$\epsilon^{-\Delta t/\tau}$			
	-00	000.00				
1	+00	00000				
	-00	00001				
2	+00	000.00	$\varepsilon = c - r$			
	-00	00000				
3	+00	00000				
	-00	00000				
4	+00	00000	$\dot{r} = \varepsilon/\tau$			
	-00	00000				
5	+00	[illegible]				
	-00	00000				
6	FST	200				
	TRA	10500	ϵ^{x} calling sequence			
7	PZE	77740				
	HTR	461				
50	CTL	600				
	TRA	77600				
1	+00	00000	3 @ 38			
	-00	00030				
2	+12	42101	1/3!			
	-10	42101				
3	+40	00000	1/2!			
	-00	00000				
4	+00	05540	1/6!			
	-26	60270				
5	+02	52521	1/4!			
	-52	52521				
6	+77	77771	.999*	one		
	+77	77771				
7	+00	00000	zero			
	-00	00000				

SM/I, Research Division, Digital Computing Facility

Program No. 125-5 Title

Programmed by: Date 5-11-61

Loc'n	Cm'd	Addr.	Contents	Accumulator	b	Remarks
. . . 60	F.S.T	. . 400	$e^{-x} = B$		. .	
. . . .	F.C.A	. . 200		$-\Delta t/\tau$	. .	
. . . . 1	X.A.R	00000			. .	
. . . .	A.L.S	00010		exp " @ 38	. .	
. . . . 2	S.T.A	77651			. .	
. . . .	A.D.D	. . 510	3 @ 38	exp + 3	. .	
. . . . 3	T.P.L	. . 020	→ exp > -3		. .	→ $x > 2^{-3}$
. . . .	C.T.V	. . 700	↓ $\Delta t/\tau \leq 1/8$		. .	calculate B directly
. . . . 4	A.D.D	. . 060	43 @ 38	exp + 46	. .	
. . . .	T.M.I	. . 070	→ exp < 40		. .	→ $x < 2^{-40}$, set B = 1
. . . . 5	X.A.R	00000			. .	
. . . .	A.R.S	00000		$-\Delta t/\tau$ @ 0	. .	↓ $2^{-3} > x > 2^{-40}$
. . . . 6	S.T.O	77770			. .	use special series
. . . .	MPR	. . 540	1/6!		. .	to calculate B
. . . . 7	C.T.L	. . 500			. .	
. . . .	A.D.D	. . 770	1/5!		. .	
. . . 70	MPR	77770	x		. .	
. . . .	ADD	77650	1/4!		. .	
. . . . 1	MPR	77770	x		. .	
. . . .	ADD	77620	1/3!		. .	
. . . . 2	MPR	77770	x		. .	
. . . .	ADD	77630	1/2!		. .	
. . . . 3	MPR	77770	x	$-x/2! + x^2/3! - + x^5/6!$	. .	
. . . .	FST	77610	temp.		. .	
. . . . 4	FCA	77660		1	. .	
. . . .	ADD	77610		$(1-e^{-\Delta t/\tau})(\tau/\Delta t)$	. .	
. . . . 5	CTL	. . 300			. .	
. . . .	FST	. . 040	B		. .	
. . . . 6	CTV	. . 100			. .	
. . . .	TRA	77700			. .	
. . . . 7	+OD	42101	1/5!		. .	
. . . .	-10	42101			. .	